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Deep Learning

- Convolutional Neural Networks 2-

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Various Types of Convolutions

Normal Convolutional Layer



1x1 Convolution (a.k.a Pointwise Convolution)

3x3 Convolution

1x1 Convolution





Separable Convolutions

• Spatial separable convolution

$$\begin{bmatrix} 3 & 6 & 9 \\ 4 & 8 & 12 \\ 5 & 10 & 15 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \times \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

Separable Convolutions

• Spatial separable convolution

Simple Convolution



Spatial Separable Convolution



Separable Convolutions

• Flattened convolution





(a) 3D convolution

(b) 1D convolutions over different directions



Flattened convolutional neural networks for feedforward acceleration, Jin et al, ICLR Workshop 2015

Depthwise Convolution



Depthwise Separable Convolution



- # parameters
 - *M*: 128 The number of input channels
 - *N*: 128 The number of output channels
 - D_K : 3 The size of a filter
 - D_F : 64 The size of a feature map



(a) Standard Convolution Filters

Standard Convolution

Depthwise Separable Convolution



(b) Depthwise Convolutional Filters



MobileNets: Efficient Convolutional Neural Networks for Mobile Vision Applications, Howard et al, CVPR 2017

- # parameters
 - *M*: 128 The number of input channels
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(a) Standard Convolution Filters

Standard Convolution

Depthwise Separable Convolution

 $\begin{array}{c}1\\D_{K} & \square & \square & \square \\D_{K} & \longleftarrow & M & \square \end{array}$

(b) Depthwise Convolutional Filters



 $D_K D_K M N$ = 147,456



- Computational Cost (FLOPS)
 - *M*: 128 The number of input channels
 - *N*: 128 The number of output channels
 - D_K : 3 The size of a filter
 - D_F : 64 The size of a feature map

Standard Convolution

Depthwise Separable Convolution

- Computational Cost (FLOPS)
 - *M*: 128 The number of input channels
 - *N*: 128 The number of output channels
 - D_K : 3 The size of a filter
 - D_F : 64 The size of a feature map

Standard Convolution

Depthwise Separable Convolution

 $D_K D_K M N D_F D_F$ = 603,979,776

 $D_K D_K M D_F D_F + M N D_F D_F$ = 71,827,456

$$\frac{D_K D_K M D_F D_F + M N D_F D_F}{D_K D_K M N D_F D_F} = \frac{1}{N} + \frac{1}{D_K^2}$$

MobileNets

ConvNet w/ depthwise separable convolution

ModelImageNetMillionMillionAccuracyMult-AddsParametersConv MobileNet71.7%486629.3MobileNet70.6%5694.2

 Table 4. Depthwise Separable vs Full Convolution MobileNet

Grouped Convolution

- Convolutions in parallel
 - It was first Introduced in AlexNet to utilize the 2 GPUs distributed training
 - 1.5GB x 2 = 3GB



Grouped Convolution

- Convolutions in parallel
 - It was first Introduced in AlexNet to utilize the 2 GPUs distributed training
 - 1.5GB x 2 = 3GB



Shuffled Convolution

- To eliminate a side effect of the grouped convolutions
 - The outputs from a certain channel are only derived from a small fraction of input channels



Shuffled Convolution

• ShuffleNet



ShuffleNet: An Extremely Efficient Convolutional Neural Network for Mobile Devices, Zhange et al, CVPR 2018

Dilated Convolution

• To aggregate multi-scale contextual information without losing resolution



Standard Convolution (I=1)

Dilated Convolution (I=2)

Review: DilatedNet — Dilated Convolution (Semantic Segmentation) | by Sik-Ho Tsang | Towards Data Science

Dilated Convolution

• To aggregate multi-scale contextual information without losing resolution



Receptive Fields

 $F_3 = \text{DilatedConv}(F_2, l = 4)$

Transposed Convolution



Transposed Convolutions explained with... MS Excel! | by Thom Lane | Apache MXNet | Medium

ImageNet Large Scale Visual Recognition Challenge

(ILSVRC)

ILSVRC

- ImageNet is an image database organized according to the WordNet hierarchy (nouns)
 - 1000 object classes
 - About 1.2M training images, 50K validation images, 100K test images
- The ImageNet Large Scale Visual Recognition Challenge (ILSVRC)
 - 8 years history (2010 2017)
 - It was the most powerful driving force to facilitate deep learning research

IM¹GENET

Classification Results



Classification Results



AlexNet

- The winner of ILSVRC 2012
- It changed the entire computer vision research



ZFNet

- The winner of ILSVRC 2013
- The network architectures were developed by using the visualization techniques
 - Visualizing and Understanding Convolutional Networks, Zeiler et al, ECCV 2014
- Reduced the 1st layer filter size from 11x11 to 7x7
- 1st layer stride from 4 -> 2

VGGNet



Architecture comparison of AlexNet, VGGNet, ResNet, Inception, DenseNet | by Khush Patel | Towards Data Science

VGGNet



Architecture comparison of AlexNet, VGGNet, ResNet, Inception, DenseNet | by Khush Patel | Towards Data Science

VGGNet

(not counting biases) INPUT: [224x224x3] memory: 224*224*3=150K params: 0 CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*3)*64 = 1,728 CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*64)*64 = 36,864 POOL2: [112x112x64] memory: 112*112*64=800K params: 0 CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*64)*128 = 73,728 CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*128)*128 = 147,456 POOL2: [56x56x128] memory: 56*56*128=400K params: 0 CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*128)*256 = 294,912 CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824 CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824 POOL2: [28x28x256] memory: 28*28*256=200K params: 0 CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*256)*512 = 1,179,648 CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2.359.296 CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296 POOL2: [14x14x512] memory: 14*14*512=100K params: 0 CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296 CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296 CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296 POOL2: [7x7x512] memory: 7*7*512=25K params: 0 FC: [1x1x4096] memory: 4096 params: 7*7*512*4096 = 102,760,448 FC: [1x1x4096] memory: 4096 params: 4096*4096 = 16,777,216 FC: [1x1x1000] memory: 1000 params: 4096*1000 = 4,096,000



TOTAL memory: 24M * 4 bytes ~= 96MB / image (for a forward pass) TOTAL params: 138M parameters

GoogLeNet

- Winner of ISLVRC 2014
- Also called 'Inception'



Max pooling

Concatenation

Convolution



GoogLeNet

• Inception module



(a) Inception module, naïve version

(b) Inception module with dimensionality reduction

GoogLeNet

• ILSVRC 2014 classification results

Team	Year	Place	Error (top-5)	Uses external data
SuperVision	2012	1st	16.4%	no
SuperVision	2012	1st	15.3%	Imagenet 22k
Clarifai	2013	1st	11.7%	no
Clarifai	2013	1st	11.2%	Imagenet 22k
MSRA	2014	3rd	7.35%	no
VGG	2014	2nd	7.32%	no
GoogLeNet	2014	1st	6.67%	no

ResNet

- The winner of ILSVRC 2015
- Residual building block



ResNet



Deep residual learning for image recognition, He et al, CVPR 2016

ResNet

• Training on ImageNet



Figure 4. Training on **ImageNet**. Thin curves denote training error, and bold curves denote validation error of the center crops. Left: plain networks of 18 and 34 layers. Right: ResNets of 18 and 34 layers. In this plot, the residual networks have no extra parameter compared to their plain counterparts.
ResNet

• ILSVRC 2015 classification results

method	top-5 err. (test)
VGG [40] (ILSVRC'14)	7.32
GoogLeNet [43] (ILSVRC'14)	6.66
VGG [40] (v5)	6.8
PReLU-net [12]	4.94
BN-inception [16]	4.82
ResNet (ILSVRC'15)	3.57

Table 5. Error rates (%) of **ensembles**. The top-5 error is on the test set of ImageNet and reported by the test server.

Wide Residual Networks (WRNs)

- Diminishing feature reuse
 - The circuit complexity theory says that shallow circuits can require exponentially more parameters than deeper ones.
 - However, in the residual block w/ identity mapping, there is nothing to force the gradients to go through residual block weights
 - It is possible that there is there is either only a few blocks that learn useful representations or many blocks share very little information with small contribution
- 'Widening' of ResNet provides a much more effective way of improving performance
 - 50 times less layers and being more than 2 times faster

Wide Residual Networks (WRNs)



Wide Residual Networks (WRNs)

• ILSVRC classification results (single crop)

Model	top-1 err, %	top-5 err, %	#params	time/batch 16
ResNet-50	24.01	7.02	25.6M	49
ResNet-101	22.44	6.21	44.5M	82
ResNet-152	22.16	6.16	60.2M	115
WRN-50-2-bottleneck	21.9	6.03	68.9M	93
pre-ResNet-200	21.66	5.79	64.7M	154

ResNeXt

- The second ranked in ILSVRC 2016
 - 'Cardinality' matters (the number of transformations in the layers)
 - next dimension -> cardinality



Aggregated residual transformations for deep neural networks, Sie et al, BMVC 2017

Ensemble of Diverse Architecture

- The winner of ILSVRC 2016
 - Trimps-Soushen Team





Review: Trimps-Soushen — Winner in ILSVRC 2016 (Image Classification) | by Sik-Ho Tsang | Towards Data Science

Ensemble of Diverse Architecture

• ILSVRC classification results

	Inception- v3	Inception- v4	Inception- Resnet-v2	Resnet- 200	Wrn-68-3	Fusion (Val.)	Fusion (Test)
Err. (%)	4.20	4.01	3.52	4.26	4.65	2.92 (-0.6)	2.99

Review: Trimps-Soushen — Winner in ILSVRC 2016 (Image Classification) | by Sik-Ho Tsang | Towards Data Science

- The feature-maps of all preceding layers are used as inputs
 - For L layers block, $\frac{L(L+1)}{2}$ direct connections



• The feature-maps of all preceding layers are used as inputs

• For L layers block,
$$\frac{L(L+1)}{2}$$
 direct connections



ResNet

DenseNet

$$\mathbf{x}_{\ell} = H_{\ell}(\mathbf{x}_{\ell-1}) + \mathbf{x}_{\ell-1}$$

$$\mathbf{x}_{\ell} = H_{\ell}([\mathbf{x}_0, \mathbf{x}_1, \dots, \mathbf{x}_{\ell-1}])$$

Densely connected convolutional networks, Huang et al, CVPR 2017

• Parameter efficient



<u>Review: DenseNet — Dense Convolutional Network (Image Classification) | by Sik-Ho Tsang | Towards Data Science</u>

• Parameter efficient



Review: DenseNet — Dense Convolutional Network (Image Classification) | by Sik-Ho Tsang | Towards Data Science

Parameter efficient



Figure 3: Comparison of the DenseNets and ResNets top-1 error rates (single-crop testing) on the ImageNet validation dataset as a function of learned parameters (*left*) and FLOPs during test-time (*right*).

- The winner of ILSVRC 2017
- SE Block
 - Easily integrated into popular convolutional modules, e.g. ResNet, Inception, etc.
 - A 'light weight' gating mechanism to model channel-wise relationships



• SE Block





SE-Inception Module



• ILSVRC classification results



	224×224		320 imes 320 /		
			299×299		
	top-1	top-5	top-1	top-5	
	err.	err.	err.	err.	
ResNet-152 [10]	23.0	6.7	21.3	5.5	
ResNet-200 [11]	21.7	5.8	20.1	4.8	
Inception-v3 [44]	-	-	21.2	5.6	
Inception-v4 [42]	-	-	20.0	5.0	
Inception-ResNet-v2 [42]	-	-	19.9	4.9	
ResNeXt-101 (64 \times 4d) [47]	20.4	5.3	19.1	4.4	
DenseNet-264 [14]	22.15	6.12	-	-	
Attention-92 [46]	-	-	19.5	4.8	
Very Deep PolyNet [51] [†]	-	-	18.71	4.25	
PyramidNet-200 [8]	20.1	5.4	19.2	4.7	
DPN-131 [5]	19.93	5.12	18.55	4.16	
SENet-154	18.68	4.47	17.28	3.79	
NASNet-A (6@4032) [55] [†]	-	-	17.3^{\ddagger}	3.8^{\ddagger}	
SENet-154 (post-challenge)	-	-	16.88 [‡]	3.58 [‡]	

The Key Ingredients of Training CNNs

Drop-out/Drop-path

Dropout

- Turning off neurons w/ given probability (e.g. 0.5)
- Every iterations, new network architectures emerge



Dropout

- A simple way to train deep neural networks for improving generalization performance
- Avoiding co-adaptations: a hidden unit cannot rely on other hidden units being present
- Model averaging



Stochastic Depth (a.k.a DropPath)

- Training short networks and use deep networks at test time
- During training, randomly drop a subset of layers and bypass them with identity function



Stochastic Depth (a.k.a DropPath)

- Linearly decaying 'drop probability'
 - Later layers will be dropped more frequently



Fig. 2. The linear decay of p_{ℓ} illustrated on a ResNet with stochastic depth for $p_0 = 1$ and $p_L = 0.5$. Conceptually, we treat the input to the first ResBlock as H_0 , which is always active.

Stochastic Depth (a.k.a DropPath)



Normalization Methods

• Normalizing training sets



• Subtracting the mean



• Divide by standard deviation



• Standardization



z-score

- When un-normalized, the loss surface is more skewed (elongated)
 - Input feature scales are very different each other

dominates the update



Both parameters can be updated in equal proportions

- Normalizing inputs (also hidden units) based on mini-batch statistics
- Computing mean and variance given the current batch
- During testing, we may not have enough batch size for this (e.g. 1 batch), using mean and variance from the training phase

$$\widehat{x}^{(k)} = \frac{x^{(k)} - \mathbf{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}$$

Input: Values of x over a mini-batch: $\mathcal{B} = \{x_{1...m}\};$ Parameters to be learned: γ , β **Output:** $\{y_i = BN_{\gamma,\beta}(x_i)\}$ $\mu_{\mathcal{B}} \leftarrow \frac{1}{m} \sum_{i=1}^{m} x_i$ // mini-batch mean $\sigma_{\mathcal{B}}^2 \leftarrow \frac{1}{m} \sum_{i=1}^m (x_i - \mu_{\mathcal{B}})^2$ // mini-batch variance $\widehat{x}_i \leftarrow \frac{x_i - \mu_{\mathcal{B}}}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}}$ // normalize $y_i \leftarrow \gamma \widehat{x}_i + \beta \equiv BN_{\gamma,\beta}(x_i)$ // scale and shift



Batch normalization: accelerating deep network training by reducing internal covariate shift, loffe et al, ICML 2015

Batch Normalization in CNN

- *M*: 128 The number of input channels
- D_F : 64 The size of a feature map
- |*B*|: 32 The mini-batch size

 $\mu \in \mathbb{R}^?, \sigma^2 \in \mathbb{R}^?$

Batch Normalization in CNN

- *M*: 128 The number of input channels
- D_F : 64 The size of a feature map
- |*B*|: 32 The mini-batch size

 $\mu \in \mathbb{R}^{128}$, $\sigma^2 \in \mathbb{R}^{128}$

Why Batch Normalization Works?

- 1. Normalization usually makes loss surface less 'skewed'
- 2. BN may reduce the internal covariance shift
 - [1502.03167] Batch Normalization: Accelerating Deep Network Training by Reducing Internal Covariate Shift (arxiv.org)
- 3. BN makes loss surface smoother
 - [1805.11604] How Does Batch Normalization Help Optimization? (arxiv.org)

Layer Normalization

- Batch normalization is dependent on the mini-batch size
 - What about the network size is too big, so only few mini-batch sizes are allowed?
- It is not obvious how to apply batch normalization to RNNs



H: the number of hidden units in a layer

Other Normalizaion Methods


Other Normalizaion Methods



Initialization

• Zero initialization



 $y = W^{(1)}x$ $z = W^{(2)}y$

• Zero initialization



$$\frac{\partial L}{\partial z} = k$$
$$\frac{\partial L}{\partial W^{(2)}} = \frac{\partial L}{\partial z} \frac{\partial z}{\partial W^{(2)}} = ky^{\mathsf{T}} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$
$$\frac{\partial L}{\partial y} = \frac{\partial L}{\partial z} \frac{\partial z}{\partial y} = kW^{(2)^{\mathsf{T}}} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
$$\frac{\partial L}{\partial W^{(1)}} = \frac{\partial L}{\partial y} \frac{\partial y}{\partial W^{(1)}} = \frac{\partial L}{\partial y} x^{\mathsf{T}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

• Same value initialization



$$\begin{aligned} \frac{\partial L}{\partial z} &= k \\ \frac{\partial L}{\partial W^{(2)}} &= \frac{\partial L}{\partial z} \frac{\partial z}{\partial W^{(2)}} = ky^{\mathsf{T}} = \begin{bmatrix} ky_1 & ky_2 & ky_3 \end{bmatrix} \\ \frac{\partial L}{\partial y} &= \frac{\partial L}{\partial z} \frac{\partial z}{\partial y} = kW^{(2)^{\mathsf{T}}} = \begin{bmatrix} kb \\ kb \\ kb \end{bmatrix} \\ \frac{\partial L}{\partial W^{(1)}} &= \frac{\partial L}{\partial y} \frac{\partial y}{\partial W^{(1)}} = \frac{\partial L}{\partial y} x^{\mathsf{T}} = \begin{bmatrix} kbx_1 & kbx_2 \\ kbx_1 & kbx_2 \\ kbx_1 & kbx_2 \end{bmatrix} \end{aligned}$$

• Same value initialization



$$\frac{\partial L}{\partial W^{(1)}} = \frac{\partial L}{\partial y} \frac{\partial y}{\partial W^{(1)}} = \frac{\partial L}{\partial y} x^{\mathsf{T}} = \begin{bmatrix} kbx_1 & kbx_2\\ kbx_1 & kbx_2\\ kbx_1 & kbx_2 \end{bmatrix}$$

 $y_1 = y_2 = y_3$

Need to break 'symmetry'

• Random initialization

 $W \sim N(0, \sigma^2)$

- Random initialization
 - Gaussian with zero mean and 'small standard deviation'
 - Gaussian distributed input data





- Random initialization
 - Gaussian with zero mean and 'small standard deviation'
 - Gaussian distributed input data





I2DL (niessner.github.io)

- Random initialization
 - Gaussian with zero mean and 'large standard deviation'
 - Gaussian distributed input data





$$y_1 = w_1 x_1 + w_2 x_2 + w_3 x_3 \cdots w_n x_n$$

- The more hidden units, less weight initial values
- Trying to 'match' the variance of each layers

$$\operatorname{Var}\left[\sum_{i=1}^{n} w_{i}x_{i}\right] = \sum_{i=1}^{n} \operatorname{Var}[w_{i}x_{i}]$$

$$= \sum_{i=1}^{n} \mathbb{E}[w_{i}^{2}x_{i}^{2}] - \mathbb{E}[w_{i}x_{i}]^{2} \qquad (\text{Independent})$$

$$= \sum_{i=1}^{n} \mathbb{E}[w_{i}]^{2}\operatorname{Var}[x_{i}] + \mathbb{E}[x_{i}]^{2}\operatorname{Var}[w_{i}] + \operatorname{Var}[x_{i}]\operatorname{Var}[w_{i}] \qquad (\text{Independent})$$

$$= \sum_{i=1}^{n} \operatorname{Var}[x_{i}]\operatorname{Var}[w_{i}] = n\operatorname{Var}[x_{i}]\operatorname{Var}[w_{i}] \qquad (\text{i.i.d})$$

- The more hidden units, less weight initial values
- Trying to 'match' the variance of each layers

$$\operatorname{Var}\left[\sum_{i=1}^{n} w_{i} x_{i}\right] = n \operatorname{Var}[x_{i}] \operatorname{Var}[w_{i}] = \operatorname{Var}[x_{i}] \qquad (\operatorname{Var}[w_{i}] = \frac{1}{n})$$

- The more hidden units, less weight initial values
- Trying to 'match' the variance of each layers

- The more hidden units, less weight initial values
- Trying to 'match' the variance of each layers

• ReLU zeros out the half of activations

$$W \sim N(0, \frac{2}{n})$$
 ReLU

• ReLU zeros out the half of activations

[docs]def xavier_normal_(tensor: Tensor, gain: float = 1.) -> Tensor: r"""Fills the input `Tensor` with values according to the method described in `Understanding the difficulty of training deep feedforward neural networks` - Glorot, X. & Bengio, Y. (2010), using a normal distribution. The resulting tensor will have values sampled from :math:`\mathcal{N}(0, \text{std}^2)` where

.. math::

```
\text{std} = \text{gain} \times \sqrt{\frac{2}{\text{fan\_in} +
\text{fan\_out}}}
```

Also known as Glorot initialization.

Args: tensor: an n-dimensional `torch.Tensor` gain: an optional scaling factor

Examples:

```
>>> w = torch.empty(3, 5)
>>> nn.init.xavier_normal_(w)
"""
```

```
fan_in, fan_out = _calculate_fan_in_and_fan_out(tensor)
std = gain * math.sqrt(2.0 / float(fan_in + fan_out))
```

```
return _no_grad_normal_(tensor, 0., std)
```

 $W \sim N(0, \frac{2}{n})$

- Deep learning is data hungry and can be easily overfitted
- Let's augment our datasets!

Data Augmentation by fastai v1. This article presents the techniques of... | by Pierre Guillou | Medium

Crop

• Cutout

• Mixup

Data (image) $\hat{x} = \lambda x_i + (1 - \lambda) x_j$ Label (one-hot) $\hat{y} = \lambda y_i + (1 - \lambda) y_j$

• Mixup

Learning Rates

Learning Rates

- The most important hyperparameter in training deep neural networks
- Even with the adaptive optimizers, e.g. ADAM, learning rate schedule is very important
- 1. The magnitude of the learning rate
 - 0.1, 0.01, 0.001, 0.0001 would be a good candidate;;
- 2. The rate of decay
- 3. Initialization

Step Decay

Learning Rate Schedules and Adaptive Learning Rate Methods for Deep Learning | by Suki Lau | Towards Data Science

Step Decay

Exponential Decay

Learning Rate Schedules and Adaptive Learning Rate Methods for Deep Learning | by Suki Lau | Towards Data Science

Cosine Scheduler

- We might not want to decrease the learning rate too drastically in the beginning
- We might want to refine the solution at the end

$$\eta_{t} = \eta_{T} + \frac{\eta_{0} - \eta_{T}}{2} (1 + \cos(\pi t/T))$$

Warmup

- Initialization is important
- By choosing sufficiently small learning rate to prevent divergence in the beginning

Activation Functions

Sigmoid (a.k.a logistic)

Saturated

$$\sigma(z) = \frac{1}{1 + \exp(-z)}$$

Tanh

Saturated

ReLU

 $\sigma(\mathbf{z}) = \max(z, 0)$

LeakyReLU

$$\sigma(z) = \max(z, 0.01z)$$

PReLU

$$\sigma(z) = \begin{cases} z, & z \ge 0 \\ az, & z < 0 \end{cases}$$

SoftPlus

$$\sigma(z) = \ln(1 + \exp(x))$$



Swish

$$\sigma(z) = \frac{x}{1 + \exp(-x)}$$

GELU

- Used in GPT3
- Recent MLP architectures

$$\sigma(z) = \frac{x}{1 + \exp(-1.702x)}$$



Sine



